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Short communication

# Background current affects the internal wave structure of the northern South China Sea

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**Abstract**

The internal wave modal equations are solved with the consideration of background currents. Analytical and numerical solutions of some specific examples, including observations in the northern South China Sea (SCS), are obtained to investigate the effect of background current on internal wave vertical structure. The effects of current shear and curvature on internal wave vertical structure are evaluated separately. It is found that the phase speed and wave structure are modified by background currents, the current shear has little effect on wave structure, whilst the current curvature could have a strong impact on the wave structure. The extent of the effect by the current curvature on the wave structure depends on the magnitudes of current curvature, relative wave speed, and buoyancy frequency, sometimes the effect by the current curvature may even cause the wave to attenuate severely with depth. A new method to obtain the real eigenfunction with depth in the case that the waves become evanescent is also put forward. It is shown that the residual tidal current in the northern SCS is strong enough to cause the wave to attenuate severely at the upper layer.


*Keywords:* Internal waves; Wave mode; Current shear; Current curvature; Numerical method; South China Sea

## 1. Introduction

Internal wave is a common phenomenon in stratified oceans. It is important during the processes of vertical and horizontal exchange of sea water and mass and heat transports, which also change the temperature and salinity field structure, the propagation of light and sound. Meanwhile, the activity of internal wave may exert important effect on floating and sinking rule [1–5]. The wave speed, modal shape and dispersion relationship are often solved by linearized wave equations [6]. Even for the analyses of nonlinear internal waves, the solution of linearized wave modal equations is often a necessary step [7]. Cai et al. [8] developed a method to estimate the forces and torques exerted by internal solitons on cylindrical piles based on the calculation of wave modal eigenfunctions, but they did not consider background currents in their calculation.

There have been studies on the kinematic and dynamic effects of background currents on the evolution of nonlinear internal wave trains [9–11]. The presence of such currents results in changes not only in the wave speed and wave form, but also in the nonlinear evolution of internal waves through the modifications of environmental parameters [7]. However, no systematic analyses have been done on the effects of current shear and curvature, i.e., the first and second derivatives of current versus depth, on the internal wave structure. Here, we evaluate these effects and demonstrate that the current curvature has a stronger effect on wave modal shape than the shear does. The actual vertical structure, not just the shear, of a background flow has to be taken into account when estimating the impact of the flow on internal waves.

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## 2. Solution of the wave modal equations

In the linear approximation, the dimensionless eigenfunctions $W_i$ (with its maximum value normalized to unity) satisfy the boundary value generalization equations [1,12]:

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\rho\frac{\mathrm{d}W_i}{\mathrm{d}z}\right)-\frac{W_i}{(c_i-u)^2}\left[(c_i-u)\frac{\mathrm{d}}{\mathrm{d}z}(\rho u')-g\frac{\mathrm{d}\rho}{\mathrm{d}z}\right]=0, \quad (1)$$

or,

$$\frac{\mathrm{d}^2W_i}{\mathrm{d}z^2}-\frac{N^2}{g}\frac{\mathrm{d}W_i}{\mathrm{d}z}+\frac{W_i}{(c_i-u)^2}\left[N^2+(c_i-u)\left(u''-\frac{N^2}{g}u'\right)\right]=0, \quad (2)$$

$$W_i=0, \quad z=0, \quad (3)$$

$$W_i=0, \quad z=D, \quad (4)$$

where $u$ is a background current, vertical shear $u'=\frac{\mathrm{d}u}{\mathrm{d}z}$, and vertical curvature $u''=\frac{\mathrm{d}^2u}{\mathrm{d}z^2}$. $N=\left(-\frac{g}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}z}\right)^{1/2}$ is the buoyancy frequency, $z$ the vertical coordinate, and $D$ the depth of the ocean. From Eqs. (2) to (4), the velocity eigenvalues $c_i$ (for mode $i$) and eigenfunctions $W_i$ can be found. For each mode, Eq. (2) can be simply written as

$$\frac{\mathrm{d}^2W}{\mathrm{d}z^2}-A\frac{\mathrm{d}W}{\mathrm{d}z}+BW=0, \quad (5)$$

where $A=\frac{N^2}{g}$, $B=\frac{N^2+(c-u)\left(u''-\frac{N^2}{g}u'\right)}{(c-u)^2}$. Under the conditions of no background current ($u=0$) and Boussinesq approximation (thus the second term of the equation is ignored), the Thompson–Haskell method [6] was used to solve Eqs. (3)–(5), where the water column is divided into many continuous levels, and if the vertical discrete level is thin enough, the buoyancy frequency $N(z)$ could be regarded as a constant so that the analytic solution of Eq. (5) can be obtained. Finally, the eigenvalues $c_i$ and their functions can be solved numerically by a joint equation transformed with a complete set of matrix equations. This method also works without Boussinesq approximation [13]. We will show examples of such calculations in the next section.

For the sake of convenience, here we focus our discussion on the case with Boussinesq approximation. Under the Boussinesq approximation, all the terms with $\frac{N^2}{g}$ in Eq. (2) could be ignored, i.e., $A\approx 0$, $B\approx\frac{N^2+(c-u)u''}{(c-u)^2}$, and thus the shear has little effect on the wave vertical structure. Suppose that the coefficient $B$ in Eq. (5) is a constant, the solution is

$$W=\begin{cases} b_1\mathrm{e}^{i\sqrt{B}z}+b_2\mathrm{e}^{-i\sqrt{B}z}, & B>0 \\ b_1\mathrm{e}^{\sqrt{-B}z}+b_2\mathrm{e}^{-\sqrt{-B}z}, & B<0 \end{cases} \quad (6)$$

where $b_1$ and $b_2$ are constants. If $B>0$, we get a cosine form solution, and the eigenfunction $W$ changes smoothly with depth. If $B<0$, we get an exponential form solution, which would cause the eigenfunction $W$ to jump sharply with depth. $B=0$ is the singular point. For the low wave modes, the phase speed is generally greater than the background current, thus the relative wave speed $(c-u)$ is greater than 0. Therefore, the eigenfunction $W$ may change largely with depth if the product of relative wave speed and current curvature is a larger negative value. We can solve the singular points for the distributions of $N$ in the case that the relative wave speed $(c-u)$ and curvature $u''$ are given (Fig. 1). It can be noted from Fig. 1 that $N$ changes within 2–13 cycle/h (cph), whose range is very common for buoyancy frequency in the stratified shelf ocean. For the specific $N$, if the current curvature varies to a large extent so that $B<0$, the eigenfunction $W$ may jump easily with depth.

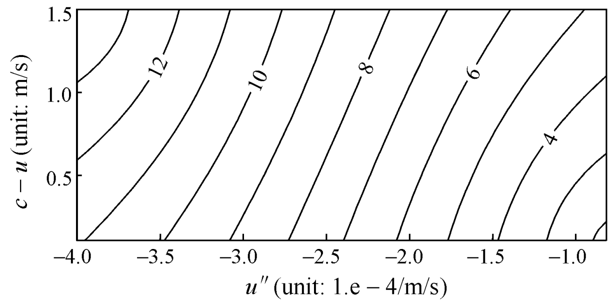


Fig. 1. Distribution of the singular points for buoyancy frequency $N$ versus parameters $(c-u)$ and curvature $u''$.

Now we discuss the possibility for the jumps appearing in the eigenfunction for the specific $u''$. When $u''$ is less than 0, it is easier for the jumps to appear in the eigenfunction of lower modes than that of higher modes. This is because $(c-u)$ is commonly greater than 0 for the lower modes, thus $B$ may be less than 0 though $u''$ keeps no change, which would cause the eigenfunction $W$ to jump with depth. Vice versa, when $u''$ is greater than 0, there are two cases: first, it is easier for the jumps to appear in the eigenfunction of higher modes than that of lower modes. This is because for the very high mode, the phase speed may turn to be lower than the current speed, thus $(c-u)$ is less than 0, and then $B$ may be less than 0 if the current curvature $u''$ has a larger positive value, and the jump in the eigenfunction may appear again. Second, since the background current $u$ changes with $z$, $(c-u)$ may be less than 0 at some depth, so that $B$ is smaller than 0 and the corresponding jump in the eigenfunction appears.

## 3. Examples and discussions

When we compare the solutions of wave modal equation with or without the current curvature, the following four simple cases are considered:

(1) Parabolic case, i.e., $u=az^2+bz+q$. Here, $a$, $b$ and $q$ are constants. Thus, the shear $u'=2az+b$, the vertical curvature $u''=2a$. For this case, the curvature in the whole depth only depends on the constant $a$.

(2) Linear case, i.e., $u = rz$. Here, $r$ is a constant. Thus, the shear $u' = r$, vertical curvature $u'' = 0$, and $B$ is also always greater than 0.
(3) No background current, i.e., $u = 0$. Thus, the shear and curvature are 0, and $B$ is always greater than 0.
(4) Observational background current.

To demonstrate that the wave vertical structure may change easily and largely due to the effect of the current curvature in reality, an example of the stratified ocean in the continental shelf of the northern South China Sea (SCS) in June, 1998, is given for discussion. The buoyancy frequency $N$ (Fig. 2) at a mooring (20° 21.311N;116° 50.633′E), with a water depth of about 470 m, was computed based on the Conductivity-Temperature-Depth observational data [8]. It has been reported that the internal solitons are very active in this sea area [8,14].

By solving Eqs. (2)–(4) using the Thompson–Haskell method [6] with a vertical resolution of 5m, we can obtain the wave phase speed and its distribution of eigenfunction for any mode, but here we only discuss phase speeds and the corresponding eigenfunctions of the first three modes and present four solutions of the real examples corresponding to the above four cases.

In case 1, suppose that the current profile (Fig. 3) changes from 0 to maximum 0.1 m/s within the upper 50 m in a symmetric parabolic form, i.e., $u = -1.6 \times 10^{-4} z(z - 50)$, with $u' = -3.2 \times 10^{-4} z$ and $u'' = -3.2 \times 10^{-4}\ \mathrm{m^{-1}\,s^{-1}}$; in case 2, the current speed changes linearly from 0.1 m/s to 0 within the upper 50 m, i.e., $u = 0.1(1 - z/50)$, with $u' = -0.002\ \mathrm{s^{-1}}$ and $u''=0$; in case 3, $u = u' = u'' = 0$; in case 4, the observational residual current at the above mooring (note that the internal soliton signal is removed), computed by the Acoustic Doppler Current Profiler (ADCP) cast data, is given as the background current. Unfortunately, the effective recording

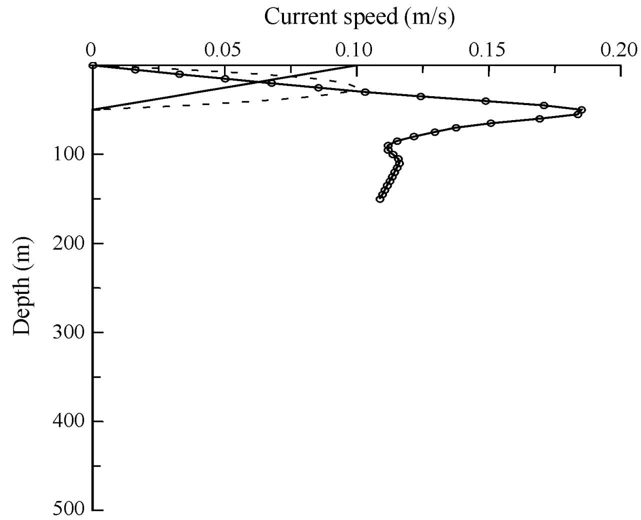


Fig. 3. Current profile of some cases (case 1: dashed line; case 2: solid line; and case 4: circled line).

depth of ADCP is only from 10 to 150 m with a 4 m resolution, thus we suppose the current speed, shear and curvature are all zero below 150 m. For current-shear cases, the local Richardson number $Ri(z) = (N/u')^2$ for wave breaking is also computed, and the resulted $Ri(z)$ is always $>1/4$, which suggests that the flow is always stable.

Fig. 4 shows the solution in case 1. The jumps in the eigenfunction appear at $z$=0–55 m for the first mode, at $z$=0–10 m for the second mode and at $z$=0–5 m for the third mode. The present wave vertical structure is very different from the normal one. In fact, the jump in the eigenfunction means that the wave is severely attenuated with depth as it propagates across this layer [1], it distorts the real eigenfunction with depth. In order to get a real solution $W$, we suppose the waves become evanescent in exponential form from an amplitude $W_0$ within the jumping

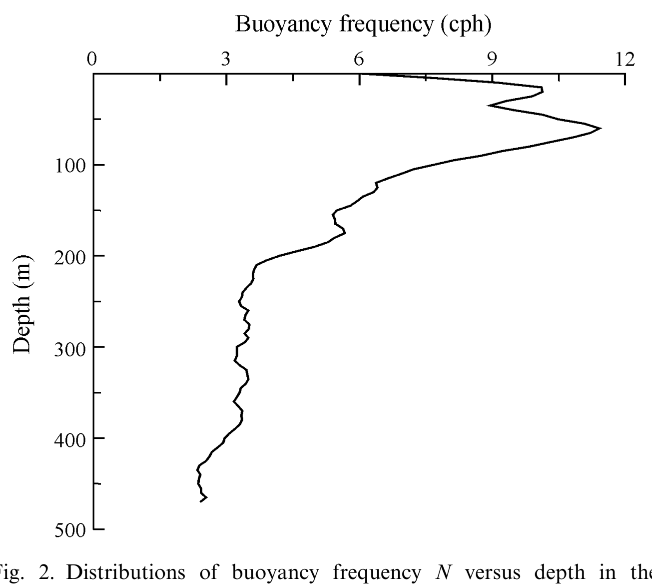


Fig. 2. Distributions of buoyancy frequency $N$ versus depth in the northern South China Sea.

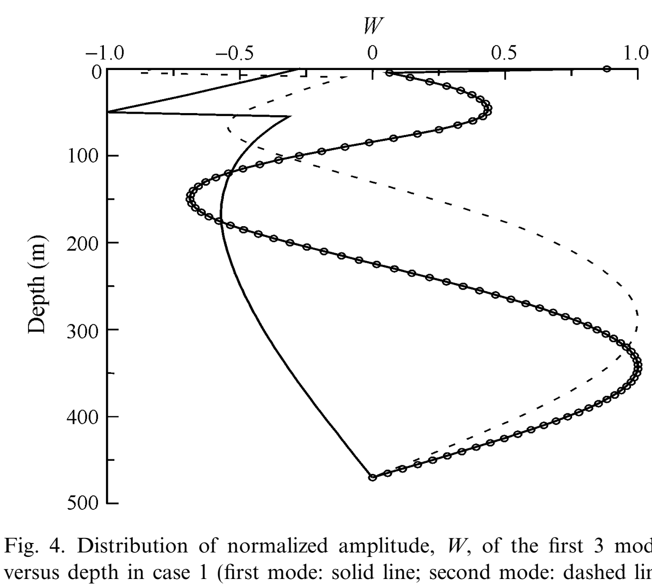


Fig. 4. Distribution of normalized amplitude, $W$, of the first 3 modes versus depth in case 1 (first mode: solid line; second mode: dashed line; and third mode: circled line).

domain (here $W_0$ is the solution at $Z_0$, the depth of the lower limit of the jumping domain. For example, $W_0$ are the solutions at 55 m for the first mode, at 10 m for the second mode and at 5 m for the third mode, respectively) instead of the present jumping form, i.e., the solution $W$ at depth $z$ where the waves become evanescent is computed as follows:

$$\begin{cases} W = b_1 e^{\sqrt{-B}z} + b_2 e^{-\sqrt{-B}z}, & 0 \leqslant z \leqslant z_0 \\ b_1 = -b_2 = W_0/(e^{\sqrt{-B}z_0} - e^{-\sqrt{-B}z_0}), & B < 0 \end{cases} \tag{7}$$

Here, $z = 0$–55 m for the first mode, $z = 0$–10 m for the second mode and $z = 0$–5 m for the third mode. Thus, in the following, we re-perform the calculations within the jump's occurring domain and only give the final real solution.

The corrected solutions of phase speeds and their corresponding eigenfunctions are shown in Table 1 and Figs. 5 and 6. Note that now no jumps appear in the solution of the eigenfunction. It is found that the difference among the phase speeds of each mode in the four cases is distinct, e.g., the phase speed of the first mode is within 1.295–1.423 m/s. The eigenfunction (i.e., the wave vertical structure) in case 1 is distinctly different from those in cases 2 and 3, especially the wave is severely attenuated within the upper 50 m for the first mode; the eigenfunctions in cases 2 and 3 are almost overlapped for the first 2 modes but are slightly different for the third mode (Fig. 5). This suggests that the effect of the current curvature on the internal wave structure is very important, which may cause the lower-mode wave to attenuate severely with depth, whilst the current shear (in case 2) has little effect on the lower-mode wave structure. In our additional experiment, we reduced the current maximum speed to 0.05 m/s (thus its curvature is also reduced) in case 1, it was found that there was no jump in the eigenfunction (figure is not shown), although it was somewhat different from that without the current curvature. This demonstrates that only when the current curvature is large enough, the severe attenuation of wave with depth could happen. Although Liu [7] considered the background current curvature in the equation of wave equations, the given current curvature may be too small to cause the severe wave attenuation with depth.

As expected, in case 4, for the adjacent mooring sea areas in the northern SCS the internal solitons are very active, the residual current is very strong, and the internal wave is attenuated severely within the upper 60 m for the first mode, and within 0–5 m for the second and third modes (Fig. 6).

Table 1
Details of the characteristic parameters of the first 3 modes for 4 cases

| Mode number | Phase speed $c$ (m/s) | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1 | 1.295 | 1.356 | 1.355 | 1.423 |
| 2 | 0.657 | 0.685 | 0.682 | 0.726 |
| 3 | 0.444 | 0.457 | 0. 452 | 0.513 |

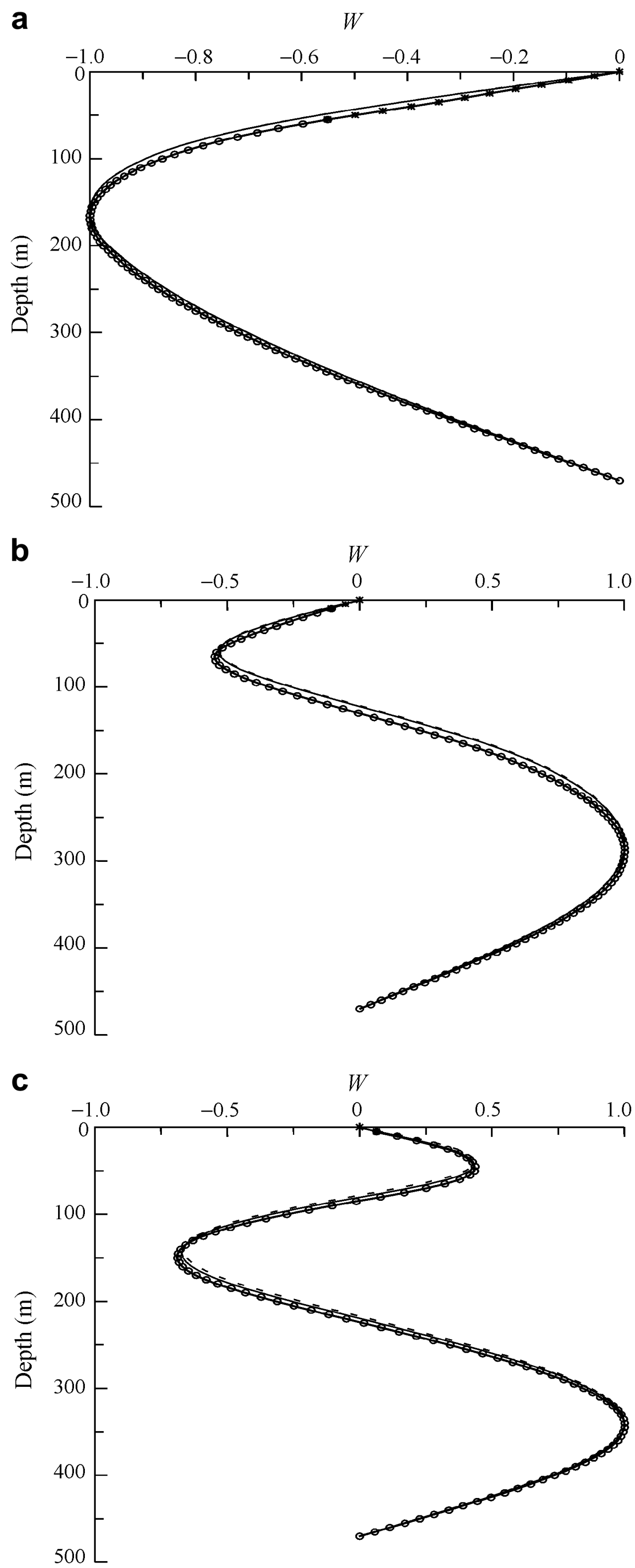


Fig. 5. Normalized amplitude ($W$) of the first mode (a), second mode (b), and third mode (c) versus depth (case 1 for corrected solution: circled line; case 2: dashed line; and case 3: solid line. Note that the corrected solutions where the waves become evanescent are marked by the asterisks).

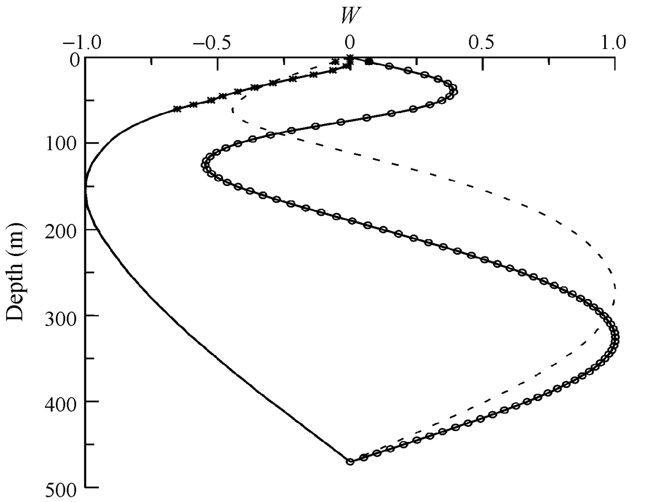


Fig. 6. Distribution of normalized amplitude ($W$) of the first 3 modes versus depth in case 4 (first mode: solid line; second mode: dashed line; and third mode: circled line. Note that the corrected solutions where the waves become evanescent are marked by the asterisks).

We can infer that since the calculation of nonlinear parameter and dispersion parameter for the evolution of nonlinear internal wave is based on the eigenfunction and its first derivative [8], their estimate of the forces and torques exerted by internal solitons on cylindrical piles would be very different if the current curvature is considered.

The current profiles in cases 1 and 4 presented here are not rare in the continental shelf sea area, in fact, the tidal residual current or wind-driven current there may be very strong at the surface, and its vertical curvature would be large, thus its effect on the wave vertical structure would be very important and cannot be ignored.

## 4. Conclusion

In this paper, based on the wave modal equations with current vertical curvature and some observational data of the northern South China Sea, the analytic and numerical solutions of some specific examples are given to demonstrate the effect of background current on internal wave vertical structure.

We have found that the phase speed and wave vertical structure are modified by background currents, whereas current shear has little effect on the wave structure. Moreover, if the product of relative wave speed and current curvature is a large negative, the effect of the current curvature on the eigenfunction with depth would be very important and cannot be ignored. It may not only affect the wave parameters such as linear wave speed, but also cause the lower-mode wave to attenuate severely with depth. The extent of the impact depends on the magnitudes of current curvature, relative wave speed and buoyancy frequency, and we also obtained the analytical results about the distribution of the singular points versus these three parameters. In addition, to obtain the real eigenfunction with depth in the case that the waves become evanescent, the eigenfunction is computed in the exponential form within the corresponding domain instead of the normal boundary condition as in Eqs. (3) and (4) in the calculations.

We also found that the observational residual tidal current in the northern SCS is strong enough to cause the wave to attenuate severely at the upper layer.

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